* 0000008000001 *

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 4 0 4 5 1 8 2 7 3 6 *

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

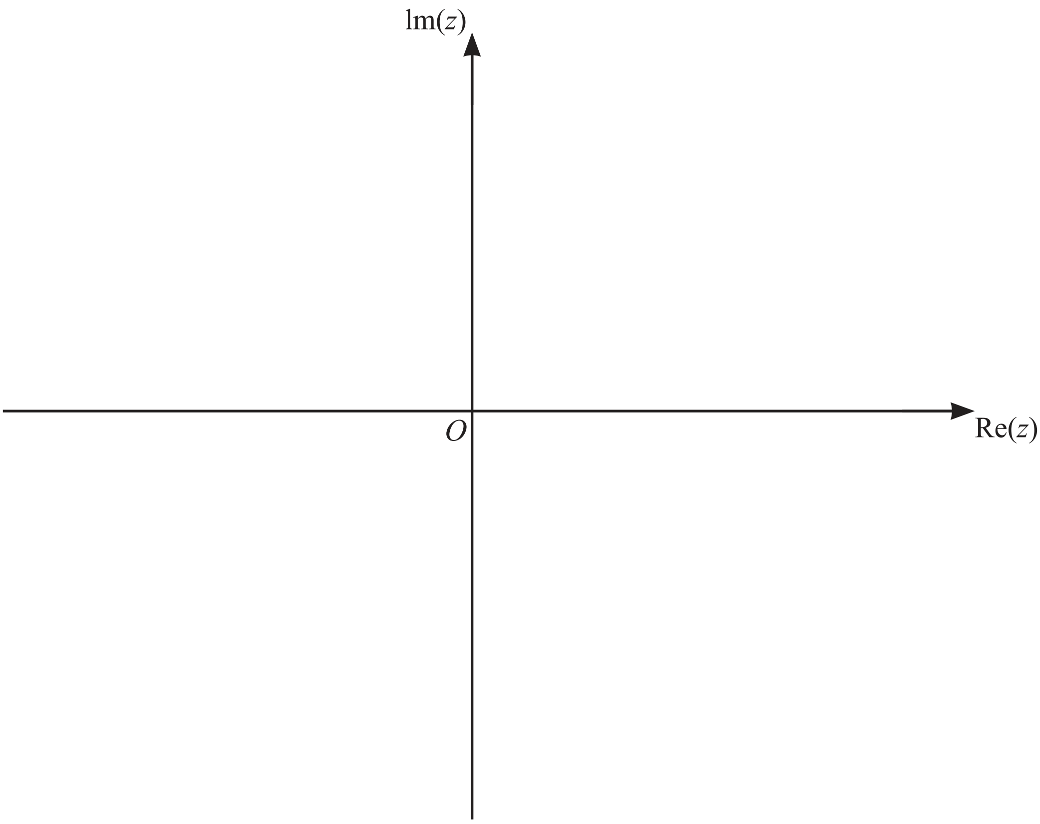
DC (DE/SG) 336976/3

[Turn over

1 The complex number z satisfies $|z| = 2$ and $0 \leqslant \arg z \leqslant \frac{1}{4}\pi$.

(a) On the Argand diagram below, sketch the locus of the points representing z. [2]

(b) On the **same diagram**, sketch the locus of the points representing z^2. [2]



2 Let $f(x) = 2x^3 - 5x^2 + 4$.

(a) Show that if a sequence of values given by the iterative formula

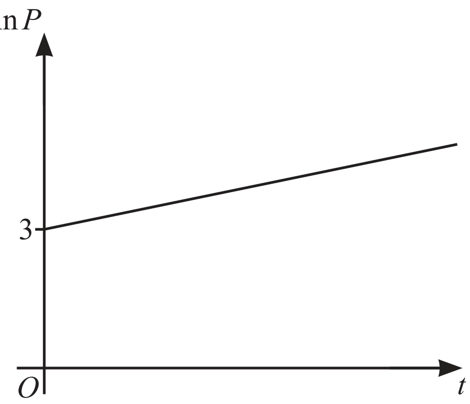
$$x_{n+1} = \sqrt{\frac{4}{5 - 2x_n}}$$

converges, then it converges to a root of the equation $f(x) = 0$. [2]

(b) The equation has a root close to 1.2 .

Use the iterative formula from part **(a)** and an initial value of 1.2 to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

3



The number of bacteria in a population, P, at time t hours is modelled by the equation $P = ae^{kt}$, where a and k are constants. The graph of $\ln P$ against t, shown in the diagram, has gradient $\frac{1}{20}$ and intersects the vertical axis at $(0, 3)$.

(a) State the value of k and find the value of a correct to 2 significant figures. [3]

....

....

....

....

....

....

....

(b) Find the time taken for P to double. Give your answer correct to the nearest hour. [2]

....

....

....

....

....

....

....

....

....

....

4 Find the complex number z satisfying the equation

$$\frac{z-3\mathrm{i}}{z+3\mathrm{i}}=\frac{2-9\mathrm{i}}{5}.$$

Give your answer in the form $x+\mathrm{i}y$, where x and y are real. [5]

5 **(a)** Show that $\cos^4\theta - \sin^4\theta - 4\sin^2\theta\cos^2\theta \equiv \cos^2 2\theta + \cos 2\theta - 1$. [3]

(b) Solve the equation $\cos^4\alpha - \sin^4\alpha = 4\sin^2\alpha\cos^2\alpha$ for $0° \leqslant \alpha \leqslant 180°$. [3]

6 The lines l and m have vector equations

$$l: \quad \mathbf{r} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{k}) \quad \text{and} \quad m: \quad \mathbf{r} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + 5\mathbf{k}).$$

Lines l and m intersect at the point P.

(a) State the coordinates of P. [1]

....

....

....

....

....

....

....

(b) Find the exact value of the cosine of the acute angle between l and m. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(c) The point A on line l has coordinates $(0, 1, 1)$. The point B on line m has coordinates $(0, 2, -8)$.

Find the exact area of triangle APB. [3]

7 The parametric equations of a curve are

$$x = 3\sin 2t, \qquad y = \tan t + \cot t,$$

for $0 < t < \frac{1}{2}\pi$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{-2}{3\sin^2 2t}$. [5]

(b) Find the equation of the normal to the curve at the point where $t = \frac{1}{4}\pi$. Give your answer in the form $py + qx + r = 0$, where p, q and r are integers. [3]

8 Let $f(x) = \dfrac{7a^2}{(a-2x)(3a+x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions. [3]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [4]

(c) State the set of values of x for which the expansion in part **(b)** is valid. [1]

9 **(a)** Find the quotient and remainder when $x^4 + 16$ is divided by $x^2 + 4$. [3]

(b) Hence show that $\int_{2}^{2\sqrt{3}} \frac{x^4+16}{x^2+4}\,dx = \frac{4}{3}(\pi+4)$. [5]

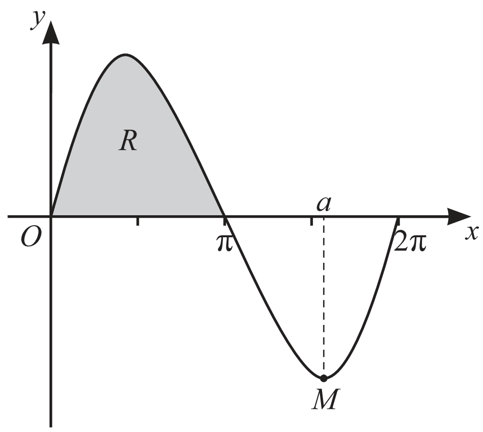
10 A water tank is in the shape of a cuboid with base area $40000\,\text{cm}^2$. At time t minutes the depth of water in the tank is h cm. Water is pumped into the tank at a rate of $50000\,\text{cm}^3$ per minute. Water is leaking out of the tank through a hole in the bottom at a rate of $600h\,\text{cm}^3$ per minute.

(a) Show that $200\dfrac{\mathrm{d}h}{\mathrm{d}t} = 250 - 3h$. [3]

(b) It is given that when $t = 0$, $h = 50$.

Find the time taken for the depth of water in the tank to reach 80 cm. Give your answer correct to 2 significant figures. [5]

11



The diagram shows the curve $y = 2\sin x\sqrt{2+\cos x}$, for $0 \leqslant x \leqslant 2\pi$, and its minimum point M, where $x = a$.

(a) Find the value of a correct to 2 decimal places. [5]

(b) Use the substitution $u = 2 + \cos x$ to find the exact area of the shaded region R. [6]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.